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# STRUCUTRE ANALYSIS LAB MANUAL

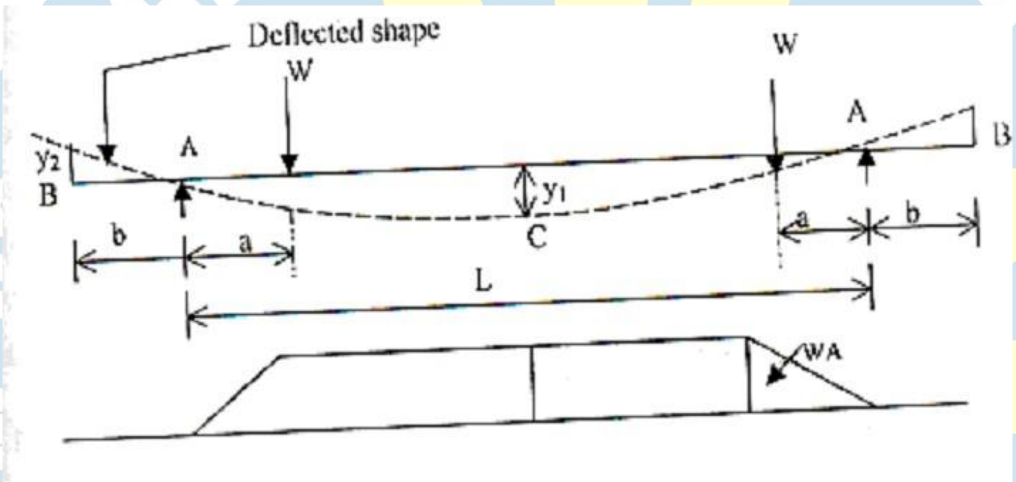
# LIST OF EXPERIMENTS

- To verify moment area theorem regarding slope and deflection in a beam
- To verify Maxwell's Reciprocal Theorem.
- Begg`sdeformeter- verification of Muller Breslau principle
- Experiment on a two – hinged arch for horizontal thrust and influence line for horizontal thrust
- Analytical and experimental study of three hinged arch
- Experimental and analytical study of unsymmetrical bending of a cantilever beam
- Sway in portal frames – Demonstration

**EXPERIMENT NO - 1**

**Aim: - To verify the moment area theorem regarding the slopes and deflections of the beam.**

**Apparatus: -** Moment of area theorem apparatus.




## Theory : -

- According to moment area theorem
- The change of slope of the tangents of the elastic curve between any two points of the deflectedbeam is equal to the area of M/EI diagram between these two points.
- The deflection of any point relative to tangent at any other point is equal to the moment of thearea of the M/EI diagram between the two point at which the deflection is required.

  Slope at B= Y2 / b

  Since the tangent at C is horizontal due to symmetry,

  Slope at B= shaded area / EI = 1 / EI [Wa2 / 2 + WA (L/2 – a)] Displacement at B with respect to tangent at C = (y1 + y2) = Moment of shaded area about B / EI

- = 1 / EI [Wa2 / 2 (b+2/3a) + Wa( L/2 –a )(b+ a/2+L/2)]

## Procedure: -

- Measure a, b and L of the beam

- Place the hangers at equal distance from the supports A and load them with equal loads.
- Measure the deflection by dial gauges at the end B (y2) and at the center C (y1)
- Repeat the above steps for different loads.

## Observation Table:-

Length of main span, L (cm) = Length of overhang on each side, a (cm) = Modulus of elasticity, E ($kg/cm^2$) = $2 \times 10^6$

| Sl . No. | Load at each Hanger (kg) | Central Deflection Y1 (cm) | Deflection at Free end y2(cm) | Slope at BY2 / b | Deflection at( Deflection at B (y1) |
|---|---|---|---|---|---|
| | | | | | |

## Calculation:

- Calculate the slope at B as y2 / b (measured value).
- Compute slope and deflection at B theoretically from B.M.D. and compare withexperimental values.
- Deflection at C = y1(measured value).
- Deflection at C = Average calculated value

**Result** :- The slope and deflection obtained is close to the slope and deflection obtained by suing moment area method.

**EXPERIMENT NO - 2**

**Aim: - To verify clerk Maxwell's reciprocal theorem**

**Apparatus: -** Clerk Maxwell's Reciprocal Theorem apparatus, Weight's, Hanger, Dial Gauge, Scale, Verniar caliper.

## Theory : -

Maxwell theorem in its simplest form states that deflection of any point A of any elastic structure due to load P at any point B is same as the deflection of beam due to same load applied at A

It is, therefore easily derived that the deflection curve for a point in a structure is the same as the deflected curve of the structure when unit load is applied at the point for which the influence curvewas obtained.

## Procedure: -

- Apply a load either at the centre of the simply supported span or at the free end of the beam, the deflected form can be obtained.
- Measure the height of the beam at certain distance by means of a dial gauge before and afterloading and

determine the deflection before and after at each point separately.

- Now move a load along the beam at certain distance and for each positions of the load, the deflection of the point was noted where the load was applied in step 1 .This deflection shouldbe measured at each such point before and after the loading, separately.
- Plot the graph between deflection as ordinate and position of point on abssica the plot for graph
- drawn in step2 and 3 .These are the influence line ordinates for deflection of the beam.

## Observation Table:-

| Distance from the pinned end | Load at central point/ cantilever end | | Deflection of various points (mm) 2-3 | Load moving along beam | | Deflection of various points (mm) 5-6 |
|---|---|---|---|---|---|---|
| | Beam unloaded Dial gauge reading (mm)2 | Beam loaded Dial gauge reading (mm)3 | Beam unloaded Dial gauge reading (mm)5 | Beam unloaded Dial gauge reading (mm)5 | Beam loaded Dial gauge reading (mm)6 | |
| | | | | | | |

**Result :** - The Maxwell reciprocal theorem is verified experimentally and analytically.

## Precaution: -

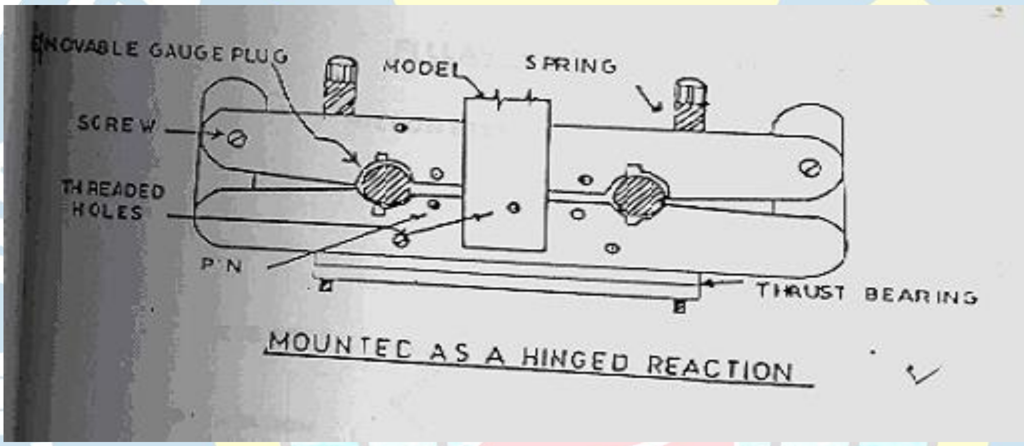
- Apply the loads without any jerk.
- Perform the experiment at a location, which is away from any
- Avoid external disturbance.
- Ensure that the supports are rigid.

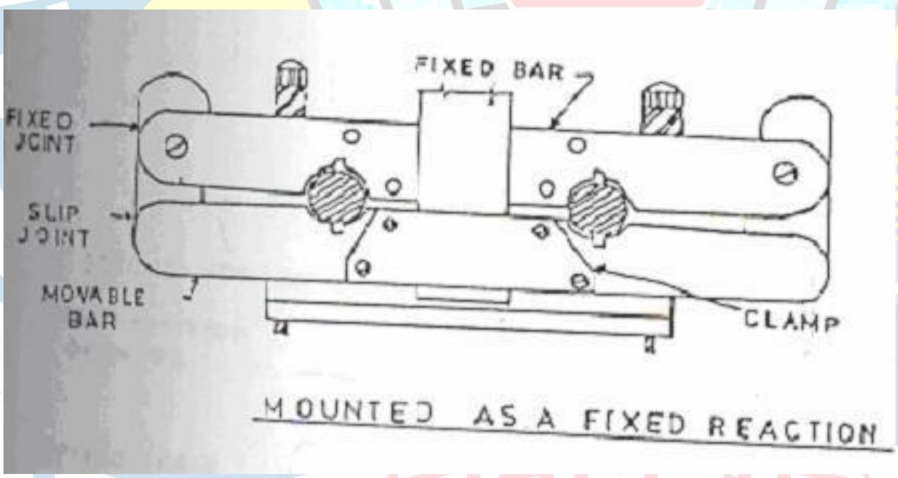
## EXPERIMENT NO - 3

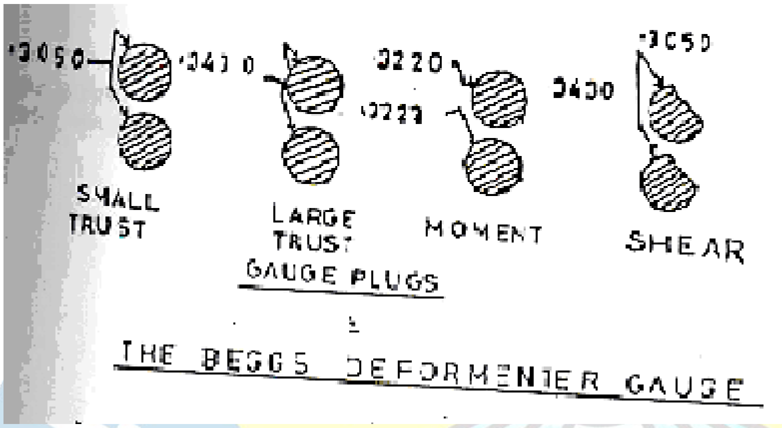
**Aim**: - To verify the Muller Breslau theorem by using Begg's deformator set.

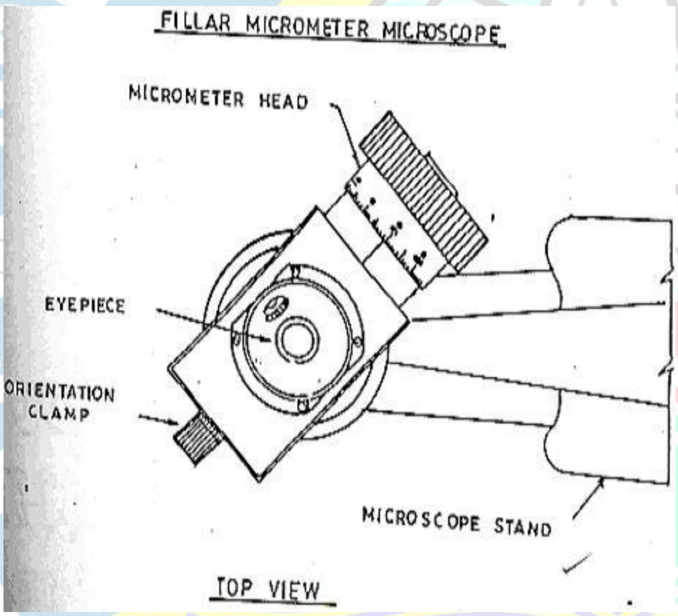
**Apparatus :**- Muller Breslau Principal Begg's Deformeter, Micrometer , Microscope,

Functioning plugs, Influence line ordinates, Drawing board , Pins etc.






**Theory :-** Various methods are available to verify Muller Breslau's Principle such as Brass wire Method, Begg's Deformeter, Eney Deformeter and Gottschalk Continostat method. The Begg's Deformeter Method is usually the most satisfactory experimental method. This principle states "the ordinates of the influence lines for any stress element ( such as axial stress, shear moment orreaction ) at any section of the structure are proportional to those of the deflection curve which is obtained by removing the restraint corresponding to that element from the structure and introducing

in its place a deformation in to the primary structure which remains.This principle is applicable to any type of structure whether statically determinate or indeterminate. Incase of indeterminate structures this principle is limited to structures, the material of which is elastic and follows Hook's law.

The application of Begg's deformeter apparatus involves the use of the relation. $F2 = kpn (\delta n/\delta 2)$

F2 = desired force component at point 2, produced by pn.pn= force assumed to be acting on structures at point n.

k= a constant; scale reduction factor if F2 is a moment component or unity if F2 is a thrustor shear component.

$\delta n$ = deformation introduced at point 2 in the direction of F2.

$\delta 2$=deformation introduced at point n caused by F2 and measured in the direction of pn

## Procedure: -

- Choose the problem for the study e.g. portal frame of equal / unequal legs, beams,trusses or any other structures.
- Choose the scale reduction factor for the linear dimensions that will give a model of asize to be made easily and used conveniently. For most structures this factor will be of such a value as to allow the model to be cut from a standard sheet.
- Select the material for model e.g. plastic sheet/Acrylic/Perspex or any other desiredsheet. The width and thickness should remain same throughout the length used.
- Cut the model in the selected shape /size .Mark the center line throughout the modellength otherwise every time one should have to find out the centre.

## Observation Table: -

**Table - 1**

Calibration of Plugs:-

| S. No | Points | Thrust | Shear | Moment | Calibration constant |
|---|---|---|---|---|---|
| | | | | | |

**Table - 2**

Influence Line Ordinates:-

| S. No | Points | Thrust | | Shear | | Moment | |
|---|---|---|---|---|---|---|---|
| | | Observed | Calculated | Observed | Calculated | Observed | Calculated |
| | 1.<br>2.<br>3. | | | | | | |

**Calculation: -** The ordinate of influence line diagram obtained experimentally:

The ordinate of influence line diagram obtained analytically:

**Result : -** The Muller Breslau Principle is verified experimentally and analytically.

**Precaution: -**

- Apply the loads without any jerk.
- Measure the deflection to the nearest of a millimeter.
- Perform the experiment at a location, which is away from any
- Avoid external disturbance.
- Ensure that the supports are rigid.

## EXPERIMENT NO - 4

**AIM:** Experiment on a 2 hinged arch for horizontal thrust and influence line for horizontal thrust.

## Theory:-

The horizontal thrust 'H' in the case of two hinged arch is given by:

$$H = \frac{\int_0^1 M_s y \, ds/E1}{\int_0^1 y^2 \, ds/E1} = \frac{\int_0^1 M_s y \, dx}{\int_0^1 y^2 \, dx}$$

Where $ds = dx \sec\theta$

$1 = 1_c \sec\theta$ and $E!_c$ is constant

Take the case of two hinged parabolic arch subjected to a concentrated load W at a distance of klfrom A. (fig. 1)

In the parabolic arch, rise of arch 'y' n at any section at a distance x from A is given by y = A x(1-x) (considering support as origin)

When $x = ½$, $y = h$, putting these valuesWe find $A = 4h/1^2$

Therefore $y = 4h/1^2 (1-x)$And $V_A = W (1-k)$

$V_B = W k$

$$\int_0^{Kl} Ms\, y\, dx = \int_0^{Kl} w\,(I-k) \times 4hx\,(1-x)dx/1^2 + \int_{kl}^{l} Wk\, 4hx\,(1-x)/1^2\, dx$$

$$= 1/3\; hWkl^2\,(l-k)\,(l+k-k^2)$$

$$\int_0^{l} y^2 dx = \int [4hx\,(l-x)/l^2]^2\, dx = 8/15\; h^2 l$$

$$H = \frac{\int Msy\, dx}{\int y^2\, dx} = \frac{1/3\; hWkl^2\,(l-k)\,(l+k-k^2)}{8/15\; h^2 l}$$

Put W = l and solve to get influence line ordinate for $H = 5\; kl/8h\,(k^3-2k^2+l)$

$H\ max. = (25/128)\,(1/x)$

## Apparatus:-

Two hinged arch model, weights, scale, dial gauge etc.

## Procedure:-

- Place the 1kg load on the first hanger position, move the lever the lever into contact with a100gm hanger on the ratio 4:1 position adjust the dia gauge to zero.
- Add 10kg to the 1kg hanger without shock and observe the dial reading.
- Restore the dial to zero reading by adding loads to the lever hanger, say the load is W.
- The experimental value of the influence ordinate at the first hanger position on is than4W/10.
- Repeat the process for all other loading position and tabulate and plot the influence ordinates.
- Compare the experimental values with lhose given by above formula.

## Precautions:-

1. Apply the load without jerk.
2. Perform the expt. Away from vibration.

## Observations and calculations:-

Span of the arch, l = mm

General rise of arch, h = mm

| Load position | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Load (kg) | | | | | | | | | | |
| Influence coordinate (4W/10) | | | | | | | | | | |
| Calculated ordinate | | | | | | | | | | |

## Results:-

| Sr. No. | Balancing Weight | Hanger Position | Horizontal Thrust Experiment<br>Wa/10b | K= positionof wt app / Span of Arch | Calculated Thrust | % crror |
|---|---|---|---|---|---|---|
| | | | | | | |
| | | | | | | |

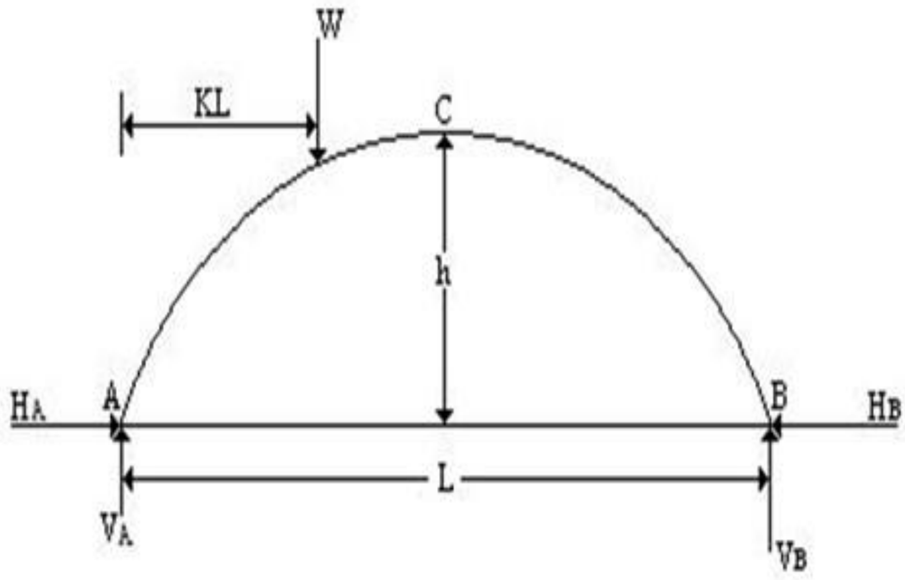



FIG. 1

## EXPERIMENT NO - 5

**Aim: - To determine the horizontal thrust in a three hinged arch for a given system of loadsexperimentally and verify the same with calculated values.**

**Apparatus: -** The model has a span of 100cm and rise 25cm with hinges at supports and chain one of the end rests on rollers. Along the horizontal span of the end rests on rollers. Along the horizontal span of the arch various points are marked at the equidistance for the application of load. A dial gauge with magnetic base is applied with the apparatus.

Theory:- A three hinged arch is a statically determination structure with the axial thrust assisting in maintaining the stability the horizontal thrust H in the arch for a number of loads can be obtained as follows:

Taking moment about Av

$R_B \times L = W1a_1 + W_2a_2 + W_3a_3$ Or $R_B = \frac{W1a_1 + W_2a_2 + W_3a_3}{L}$

Taking moment about B

$R_A = \frac{W_1 (L-a_1) + W_2 (L - a_2) + W3 (L-a_3)}{L}$

Taking the moment of all the forces on left hand side about C, wt getH X Y +W1 $(L/2 - a_1) + W_2$

$(L/2 - a_2) + R_A \times L/2$

$H = 1/r [ 1/2 [ W_1 (L-a_1) + W_2 (L-a_2) + W_3 (L-a_2) - W_1 (/2/L-a_1) - W_2 (L/2-a_1) \}]$

The value of horizontal reaction can be evaluated by Equ. (1)

The influence line of any reaction at a point is a graph showing the variation of functions, moments, shear forcers, stress at deflections at a point for various positions of unit moving load. Therefore to draw the influence lone for H, a unit load of 14g is placed the influence lone for H, a unit load of 14g is placed at varying distance from either of the supports.

Let a load of 1kg be placed at a distance from A $R_B = X/L$ then, taking moment about for all the forces on R.H.S. of c we have

$= x/2 \times l/2 - Hr = 0$

$H = x/2r$ ........................(2)

Thus the equation (2) is the equ. Of a straight line and gives the influence line diagram of thehorizontal reaction.

## Procedure: -

- Use lubricating oil the roller end of the arch so as to have a free movement of the roller end. Balance the self weight of the arch by placing load on the hanger for horizontal thrust until the fest equilibrium conditions are obtained. Note down the loads in kgs.
- Place a few loads on the arch in any chosen positions. Balance these by placing additional weights on the hanger for horizontal thrust. The additional weights on the thrust hanger give the experimental value of the horizontal thrust.
- To obtain the influence line for H, place a load of 2kg in twin on each hanger one by one and find the balancing weights required on the thrust hanger.
- Plot the ordinate representing 1/2 of the balancing weights on the load positions as base. This gives the influence line diagram for horizontal thrust.

## Calculation: -

Span of the arch, L =100cmCentral rise, h =25cm

Initial load on the thrust hanger for balancing kg =1.5kg

## Precaution: -

- Plot weight in thrust hanger very gently without a jerk.
- Measure the distance of loaded points from left hand support accurately.
- Perform the experiment away from vibration and other disturbance.

# EXPERIMENT NO – 6

**Object:-**Experimental and analytical study of defections for unsymmetrical bending of a cantilever beam.

## Theory:-

In structural members subjected to flexure, the Euler Bernoulli's equation $\{\sigma/y = M/1 = E/R\}$ is valid only if the applied bending moment acts about one or the other principal axis of the cross section. However a member may be subject to a bending moment which acts on a plane inclined to the principal axis (say). This type of bending does not occur in a plane of symmetry of the cross section, it is called unsymmetrical bending. Since the problem related to flexure in general differs from symmetrical bending, it may be termed as skew bending.

Every cross-section of a symmetric section has two mutually perpendicular principal axes of inertia, about one of which the moment of inertia is the maximum and about the other a minimum. It can be shown that the symmetric axis of cross-section is one of the principal axesand one at right angles to the same will be the other principal axis.

From the principal of mechanics, any couple which may cause bending moment at a section of a beam may be resolved in to two components. The component of the bending moment acting around x is M cos α, while the one acting around the y-axis is M is α. The sense of each component follows from the sense of the total moment M. These moments may be used separately in the usual flexure formula and the compound normal stresses follow by super position as follows:

$\sigma = \pm M_{xx}.y/1_{xx} \pm M_{yy}.x\,/1_{yy}$ ............................................................................................(i)

One or the other of the principal axes of the cross-section.
Bending Moment in a plane that is not coincident with either of the principal axes.

Components of the bending moment in the plane of the principal axes.

For beams having unsymmetrical cross-section such as an angle (L) or a channel ([) section, if the plane of loading is not coincident with or parallel to one of the principal axes, the bending is not simple. In that case it is said to be unsymmetrical or non-uniplanar bending.

In the present experiment for a cantilever beam of an angle section, the plane of loadingis always kept vertical and the angle iron cantilever beam itself is rotated through angles in steps of 45°.

Consider the position of the angle section as shown in Fig. (2). The plane of loading makes an angle ϕ with V-V axis of the section, which is one of the principal axes of the section. The components of the vertical load P along V-V and U-U axes are P cos ϕ and P sin ϕ respectively. The deflections U and V along U-V and V-V axes respectively are given by

$$\Delta_y = P\sin\varphi .\ L^3/3EI_{vv} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

$$\Delta_y = P\cos\varphi .\ L^3/3EI_{uu} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(iii)}$$

And the magnitude of resultant deflection $\Delta_{00}$, is given by

$$\Delta = \sqrt{\Delta^2 y + \Delta^2_{\varsigma}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(iv)}$$

And its direction is given by

$$\beta = \tan^{-1} \Delta_{\varsigma} / \Delta_{y} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(v)}$$

Where β is the inclination of the resultant deflection with the U-U axes. This resultant displacement is perpendicular to the neutral axis n-n (Fig. 3) but notin the plane of the load P. InFig. the following notation has been used:-

00’ = D

0’p = $D_V$

0P = $D_U$

0’Q = $D_X$

$0Q = D_Y$

$$\Delta_y = \frac{P\cos\varphi .\ L^3}{3EI_{uu}}$$

$\tan\beta = \Delta_\varsigma / \Delta_y = 0'P/0P =$

$$\Delta_y = \frac{P\sin\varphi .\ L^3}{3EI_{vv}}$$

$= I_{vv} / I_{uu} \cot\varphi$..................................................................(vi)

For the angle section used in the present experiment $I_{uu}$ and $I_{vv}$ can be known from the tables of Bureau of Indian Standard hand book for properties of standard section. Therefore a given angle ϕ, the magnitude of angle β can be calculated from equation (vi).

The horizontal and vertical components of the deflection can be calculated on the basis ofthe geometry available as shown in Fig. 4. It can be seen.

$\Delta_\zeta = \Delta. \cos(\varphi + \beta)$

> ..............................(vii)

$\Delta_\psi =$

$\Delta. \sin(\varphi +$

$\beta)$Similarly

$\Delta_\zeta = \Delta. (\cos\varphi \cos\beta - \sin\varphi \sin\beta)$

$= \Delta_y \cos\varphi\ \Delta_\zeta \sin\beta$...................................(viii)

$\Delta_\psi = \Delta. (\sin\varphi \cos\beta + \cos\varphi \sin\beta)$

$= \Delta_y \sin\varphi\ \Delta_\zeta \cos\beta$...................................(xi)

Therefore the procedure of calculating the deflections would be:-

- Calculate $\Delta_U$ and $\Delta_V$ using equations (ii and iii)
- Compute $\Delta$ using equations (iv)
- Compute $\beta$ using equation (vi)
- Calculate the required values of $\Delta_x$ $\Delta_y$ using equations (viii) and (ix).

## Apparatus:-

- Cantilever beam having an equal angle section. The beam is fixed at one end with possibility of rotation of 45° intervals and clamped. At the free end, the loading arrangements are such that vertical loading is always ensured, dial gauges, weights etc.

## Procedure:-

- Clamp the beam at zero position and put a weight of 500gms (5N) on the hanger and lakethe zero loading on the beam to activate the member.
- Set the dial gauges to zero reading to measure vertical and horizontal displacements at the free end of the beam.
- Load the beam in steps of 1kg (10N) up to 8kg (80N) and note the vertical and horizontal deflections each time.
- Repeat the steps (a) to (c) turning the beam through 45° intervals. Problem of unsymmetrical bending will arise only in those cases where the legs of the angle section are in horizontal and vertical positions. In those cases both vertical and horizontal deflections need be measured.
- Compute the theoretical deflections and compare with those measured experimentally.

## Precautions:-

- Take care to see that you do not exert force on the free end of the cantilever beam.
- Put the load on the hanger gradually without any jerk.
- Perform the test at a location which is free from vibrations.

## Observations and calculations:-

Material of beam – mild steel.

Younges modulus of the material (E) = $2 \times 10^6$ kg/cm² ($2\times10^5$ N/mm²)

Span of cantilever beam (L) = cm

Sectional properties

Size =

ii. $I_{xx}$ = $mm^4$

iii. $I_{yy}$ = $mm^4$

iv. $I_{UU}$ = $mm^4$

v. $I_{vv}$ = $mm^4$

vi. Area = $mm^2$

# EXPERIMENT NO – 7

**Object:-Sway in portal frames-demonstration.**

**Theory:-**Sway in portal frames may be due to the following reasons:-

- Eccentric or unsymmetrical loading on the portal frame
- Unsymmetric outline of portal frame .
- Different end condition of the cohimns of the portal frames
- Non-Uniform section of the members of the frames
- Horizontal loading of the supports of the frames
- Settlement of the supports of the frames
- A combination of the above

In such cases, the joint translations become additional unknown quantities. Some additional conditions will, therefore, be required for analyzing the frames. The additional conditions of equilibrium are obtained from the consideration of the shear force exerted on the structure by the external loading. The horizontal shear exerted by a member is equal to the algebraic sum of the moments at the ends divided by the length of the members. Thus the horizontal shear resistance of all such members can be found and the algebraic sum of all such forces must balance the external horizontal loading. If any, see Fig. (viii)

Taking moments about top joint, expressions for horizontal shear at supports A and B will be

$$H_A = \frac{M_{AB} + M_{BA} - Pa}{L_{AB}}$$

$$H_D = \frac{M_{DC} + M_{CD}}{L_{CD}}$$

After knowing $H_A$ and $H_D$, we can write horizontal equilibrium equation for the frame as $H_A + H_D + P = 0$

This gives additional equation required. It is called shear equation/shear condition. Hence $Q_A$ and $Q_B$ and $\Delta$ (Sway) can be found.

Similarly for case given below (sec. fig. ix)

The shear equation can be written as $H_A + H_D + P = 0$

$$H_A = \frac{M_{BA} + M_{AB} - P_a / L}{L}$$

$$H_P = \frac{M_{CD} + M_{DC}}{L}$$

## Apparatus:-

Portal frames model, weights, scale and gauges etc.

## Procedure:-

- First of all confirm about the support conditions.
- Note down the initial reading on the gauges and confirm that frame is horizontal prior to loading.
- Apply load care fully.
- Note down the horizontal deflection with the help of gauges.
- Calculate theoretically the sway by either slope defection method or moment distribution method.
- Compare both values and find out error.
- Repeat the procedure for different loading conditions.